dary. At depths when the dynamic scattering becomes essential this peak splits into a number of peaks arranged symmetrically with respect to the boundary (Fig. 5).

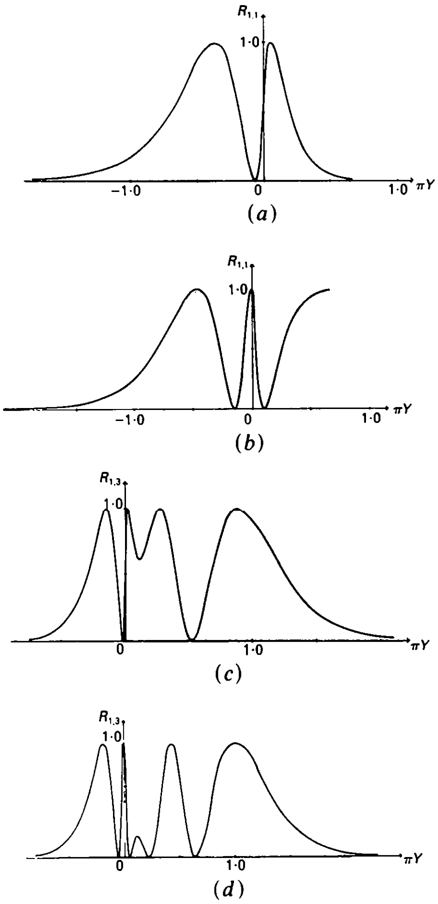

Fig. 7. Dynamical profiles for $l=1$. (a) $n=1$, $A=2$; (b) $n=1$, $A=2\cdot5$; (c) $n=3$, $A=2$; (d) $n=3$, $A=2\cdot5$.

5. Concluding remarks

The basic rules obtained in considering X-ray diffraction by a crystal with a twist boundary perpendicular to the crystal surface make it possible to investigate the structure of the boundary-side layer of bicrystals. The direct-image method (Amelinckx & Dekeyser, 1959) and the diffraction-image method suggested in this work complement each other, but the latter makes it possible to perform a more detailed study of the boundary-side layer structure. In the present work we did not take into account wave-field absorption and the incident-beam divergence. Absorption is important for thick crystals and will cause softening of the *Pendellösung* fringe contrast due to the anomalous absorption effect. The incident beam divergence, essential in the X-ray case, leads to the consideration of an incident spherical wave. In this case the form and the arrangement of the *Pendellösung* fringes will be different.

It can be shown that if there is a deviation from the periodicity in SL, the situation is analogous to the thermal diffuse scattering of X-rays by crystals and can be described by an analog of the Debye-Waller factor.

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A New Method of Deriving and Cataloguing Simple and Multiple Antisymmetry G_3^l Space Groups

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Abstract

Antisymmetric characteristics of 230 (Fedorov) G_3 space groups are constructed. By application of a partial cataloguing method based on a newly defined term of antisymmetric characteristic type, a partial catalogue of simple and multiple antisymmetry M^m-type space groups and numbers of these groups are obtained.

The G_3^l space groups of simple and multiple antisymmetry, derived from the 230 Fedorov G_3 space groups,

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their cataloguing and number N_m of M^m-type group computations have been discussed in detail by Belov, Neronova & Smirnova (1955, 1957), Zamorzaev (1957, 1962, 1963, 1976), Zamorzaev & Palistrant (1964, 1980), Zamorzaev, Palistrant & Galjarski (1978) and Jablan (1986*b*). The derivation and cataloguing problems of G_3^l groups (Shubnikov generalized or Zamorzaev groups) of M^m type for $m=1$ and $m=2$ were solved by Zamorzaev & Palistrant. Their results are published in the form of a complete catalogue of M^1-type antisymmetry space groups (Zamorzaev, 1976, Table *P*2) and a partial catalogue of M^2-type groups (Zamorzaev, Palistrant & Galjarski, 1978, Table *P*5), by class representatives. $N_1=1191$ M^1-type groups and $N_2=9511$ M^2-type groups are derived.

Because of the large volume of results, the derivation process of the M^m-type groups $m\geq 3$ is partially realized, in certain classes only, so the $N_m(G)$ numbers for $m=3$, 4, 5 remain uncomputed. Besides the mentioned results, obtained by cataloguing, the $N_q(G)$ numbers of M^q-type antisymmetry groups, derived from Fedorov G groups, where q is the maximum simple or multiple antisymmetry for which the G group generates the M^m-type groups, can be computed by combinatoral methods. The numerical results are given by Zamorzaev & Palistrant (1964) and Zamorzaev (1976, Table III_2). Because the M^6-type groups are derived exclusively from the *Pmmm*(18*s*) symmetry group, the result $N_6(Pmmm)=N_6(18s)=N_q(Pmmm)=419\,973\,120$, obtained by the combinatoral method, is also the number N_6 of all Zamorzaev M^6-type groups. For $m>6$, the Fedorov groups do not generate multiple antisymmetry M^m-type groups, so for $m>6$ all numbers N_m are equal to zero.

The universal method of derivation (Jablan, 1984, 1986*b*), partial cataloguing and computation of N_m and $N_m(G)$ numbers of M^m-type antisymmetry groups (Jablan, 1986*b*), based on the newly defined terms: the antisymmetric characteristic of a symmetry group and the antisymmetric characteristic type, is applied in this study to the construction of the antisymmetric characteristics of 230 Fedorov groups, derivation of corresponding simple and multiple M^m-type groups, their partial cataloguing and computation of $N_m(G)$ and N_m numbers.

From theoretical suppositions (Jablan, 1984, 1986*a*), the antisymmetric characteristics of the 230 Fedorov groups are formed. As the symmetry groups that possess isomorphic antisymmetric characteristics generate the same number of M^m-type antisymmetry groups for every fixed m, which correspond to each other with regard to structure (Jablan, 1984, 1986*a*, Theorem 4), only of interest is the derivation of simple and multiple antisymmetry M^m-type groups from the symmetry groups with nonisomorphic antisymmetric characteristics. According to the isomorphism relationship of antisymmetric characteristics, the 230 Fedorov symmetry groups can be distributed in the 34 equivalency classes (I-XXXIV). Therefore, the derivation of simple and multiple antisymmetric G_3 space groups is reduced to the derivation of the simple and multiple antisymmetry M^m-type groups from the 34 Fedorov-group representatives of the I-XXXIV classes, where in each class any group (*e.g.* the first) can be a representative of the class (Jablan, 1986*b*).

In Table 1 the data are given in the following order: the cardinal number of the equivalency class (I-XXXIV), the cardinal number of the Fedorov group in symbols (Zamorzaev, 1976, Table *P*1), the international symbol, the presentation in symbols (Zamorzaev, 1976, Table *P*1) of the reduced antisymmetric characteristic and the condition that results from the existential criterion for the M^m-type antisymmetry groups (Jablan, 1984, 1986*b*, Theorem 1*a*). The prohibition of substitution of generator g by antigenerators is marked by $g\neq\bar{g}$ and the obligation of simultaneous substitution of g, g_1 generators by antigenerators of identical antisymmetry by $g=g_1$. The data are given for each of the I-XXXIV classes.

The derivation of the simple and multiple antisymmetry M^m-type groups in the XIX, XX, XXIII and XXIV classes is trivial and can be carried out using the criterion of independence of the $e_1, e_2, \ldots, e_m$ antiidentities (Jablan, 1984, 1986*a*, Theorem 1*b*) for groups of the M^m-type, but in the XV class, according to the existential criterion for M^m-type groups (Jablan, 1984, 1986*b*, Theorem 1*a*), the derivation of simple and multiple antisymmetry M^m-type groups cannot be effected.

The second phase of the derivation of the simple and multiple M^m-type G_3^l space groups is based on a newly defined concept: the antisymmetric characteristic type. The solution is based on the following theoretical suppositions.

Definition 1: Let the G symmetry group, with its (reduced) antisymmetric characteristic $AC(G)$, which consists of subsets of transformations equivalent with respect to symmetry, be given. Each of these subsets is the segment of degree 1 of $AC(G)$. Each segment of $AC(G)$ which consists of two or more of the n degree segments is an $n+1$ degree segment ($n\geq 1$).

Definition 2: Let an arbitrary segment of degree 1 of antisymmetric characteristic $AC^m(G_i)$ of any simple or multiple antisymmetry M^m-type group G_i^m be given. The type of this segment is the number of different segments of degree 1 which are obtained from the segment by transition from m antisymmetry to $m+1$, in the process of deriving the M^{m+1}-type groups from the G_i^m group. The type of $n+1$ degree segment is a combination of the types of n degree segments, of which the $n+1$ degree segment consists. The combination of all types of segments of which

Table 1. *The reduced antisymmetric characteristics of the 230 Fedorov groups*

I	1*s*	*P*1	$\{a, b, c\}\ \{a, b, c, ab, ac, bc\}$	
II	2*s*	$P\bar{1}$	$\{a, b, c\}(\bar{2})\ \{\bar{2}, \bar{2}a, \bar{2}b, \bar{2}c, \bar{2}ab, \bar{2}ac, \bar{2}bc, \bar{2}abc\}$	
III	3*s*	*P*2	$\{a, b, c\}(2)\ \{c\}\{2, 2a, 2b, 2ab\}$	
	2*a*	$P2_1/m$	$\{a, b, c\}\left(\frac{c}{2}2:m\right)\ \{m\}\left\{\frac{c}{2}2, \frac{c}{2}2a, \frac{c}{2}2b, \frac{c}{2}2ab\right\}$	$c \neq \bar{c}$
IV	4*s*	*B*2	$\left\{a, b, \frac{a+c}{2}\right\}(2)\ \{2, 2b\}\left\{\frac{a+c}{2}2, \frac{a+c}{2}2b\right\}$	$a \neq \bar{a}$
	26*s*	$P\bar{4}$	$\{a, b, c\}(\bar{4})\ \{\bar{4}, \bar{4}b\}\{\bar{4}c, \bar{4}bc\}$	$a = b$
	1*h*	*Pb*	$\{a, b, c\}\left(\frac{b}{2}m\right)\ \left\{\frac{b}{2}m, \frac{b}{2}ma\right\}\left\{\frac{b}{2}mc, \frac{b}{2}mac\right\}$	$b \neq \bar{b}$
	33*h*	$P\bar{4}n2$	$\{a, b, c\}\left(\bar{4}\frac{b+c}{2}m_{a/4}\right)\ \left\{\frac{b+c}{2}m_{a/4}, \frac{b+c}{2}m_{a/4}a\right\}\{\bar{4}, \bar{4}a\}$	$a = b = c$
	3*a*	$P2_1/b$	$\{a, b, c\}\left(\frac{c}{2}2:\frac{b}{2}m\right)\ \left\{\frac{c}{2}2, \frac{c}{2}2a\right\}\left\{\frac{c}{2}2\frac{b}{2}m, \frac{c}{2}2\frac{b}{2}ma\right\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	7*a*	$P2_122_1$	$\{a, b, c\}\left(\frac{c}{2}2:\frac{a}{2}2\right)\ \left\{\frac{c}{2}2, \frac{a}{2}2\right\}\left\{\frac{c}{2}2b, \frac{a}{2}2b\right\}$	$a \neq \bar{a}\ c \neq \bar{c}$
	42*a*	$P4_2/n$	$\{a, b, c\}\left(\frac{c}{2}4:\frac{a+b}{2}m\right)\ \left\{\frac{c}{2}4, \frac{c}{2}4a\right\}\left\{\frac{c}{2}4\frac{a+b}{2}m, \frac{c}{2}4a\frac{a+b}{2}m\right\}$	$a = b = c$
V	5*s*	*Pm*	$\{a, b, c\}(m)\ \{a, b, ab\}\{m, mc\}$	
VI	6*s*	*Bm*	$\left\{a, b, \frac{a+c}{2}\right\}(m)\ \{m\}\left\{\frac{a+c}{2}, b\frac{a+c}{2}\right\}$	$a \neq \bar{a}$
	16*s*	*Imm*2	$\left\{a, b, \frac{a+b+c}{2}\right\}(2m)\ \left\{\frac{a+b+c}{2}\right\}\{m, 2m\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	22*s*	*P*4	$\{a, b, c\}(4)\ \{c\}\{4, 4a\}$	$a = b$
	35*s*	$I\bar{4}m2$	$\left\{a, b, \frac{a+b+c}{2}\right\}(\bar{4}m)\ \{m\}\left\{\bar{4}, \bar{4}\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	55*s*	$P\bar{3}m1$	$\{(a, b), c\}(3m:2)\ \{m\}\{2, 2c\}$	$a \neq \bar{a}\ 3 \neq \bar{3}$
	56*s*	$P\bar{3}1m$	$\{(a, b), c\}(2:m3)\ \{m\}\{2, 2c\}$	$a \neq \bar{a}\ 3 \neq \bar{3}$
	57*s*	$R\bar{3}m$	$\{a, b, c\}(2:m3)\ \{m\}\{2, 2c\}$	$a = b = c\ 3 \neq \bar{3}$
	47*s*	$P\bar{6}2m$	$\{(a, b), c\}(3:m2)\ \{m\}\{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	48*s*	$P\bar{6}m2$	$\{(a, b), c\}(2m:3)\ \{m\}\{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	53*s*	$P\frac{6}{m}$	$\{(a, b), c\}(6:m)\ \{6\}\{m, cm\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	54*s*	*P*622	$\{(a, b), c\}(6:2)\ \{6\}\{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	71*s*	*Pm*3*m*	$\{a, b, c\}(3/4m)\ \{m\}\{4, 4a\}$	$a = b = c\ 3 \neq \bar{3}$
	4*h*	$B\frac{2}{b}$	$\left\{a, b, \frac{a+c}{2}\right\}\left(2:\frac{b}{2}m\right)\ \{2\}\left\{\frac{b}{2}m, \frac{b}{2}m\frac{a+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	7*h*	*Pnc*2	$\{a, b, c\}\left(2\frac{b+c}{2}m\right)\ \left\{\frac{b+c}{2}m\right\}\{2, 2a\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	9*h*	*Pba*2	$\{a, b, c\}\left(2\frac{b}{2}m_{a/4}\right)\ \{c\}\left\{\frac{b}{2}m_{a/4}, 2\frac{b}{2}m_{a/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	10*h*	*Ccc*2	$\left\{a, \frac{a+b}{2}, c\right\}\left(2\frac{c}{2}m\right)\ \left\{\frac{a+b}{2}\right\}\left\{\frac{c}{2}m, 2\frac{c}{2}m\right\}$	$a \neq \bar{a}\ c \neq \bar{c}$
	15*h*	*Iba*2	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(2\frac{c}{2}m\right)\ \left\{\frac{a+b+c}{2}\right\}\left\{\frac{c}{2}m, 2\frac{c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	25*h*	*P*4*cc*	$\{a, b, c\}\left(4\frac{c}{2}m\right)\ \left\{\frac{c}{2}m\right\}\{4, 4a\}$	$a = b\ c \neq \bar{c}$
	29*h*	$P\frac{4}{n}$	$\{a, b, c\}\left(4:\frac{a+b}{2}m\right)\ \{4\}\left\{\frac{a+b}{2}m, \frac{a+b}{2}mc\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	30*h*	$P\bar{4}2c$	$\{a, b, c\}(\bar{4}_{c/4}:2)\ \{2\}\{\bar{4}_{c/4}, \bar{4}_{c/4}a\}$	$a = b\ c \neq \bar{c}$
	31*h*	$P\bar{4}c2$	$\{a, b, c\}\left(\bar{4}\frac{c}{2}m\right)\ \left\{\frac{c}{2}m\right\}\{\bar{4}, \bar{4}a\}$	$a = b\ c \neq \bar{c}$
	32*h*	$P\bar{4}b2$	$\{a, b, c\}\left(\bar{4}\frac{b}{2}m_{a/4}\right)\ \left\{\frac{b}{2}m_{a/4}\right\}\{\bar{4}, \bar{4}c\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	34*h*	$I\bar{4}c2$	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(\bar{4}\frac{c}{2}m\right)\ \left\{\frac{c}{2}m\right\}\left\{\bar{4}, \bar{4}\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	5*a*	$C222_1$	$\left\{a, \frac{a+b}{2}, c\right\}\left(\frac{c}{2}2:2\right)\ \left\{\frac{a+b}{2}\right\}\left\{2, 2\frac{c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	10*a*	$Pnm2_1$	$\{a, b, c\}\left(\frac{c}{2}2\frac{b+c}{2}m\right)\ \left\{\frac{b+c}{2}m\right\}\left\{\frac{c}{2}2, a\frac{c}{2}2\right\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	11*a*	$Pbc2_1$	$\{a, b, c\}\left(\frac{c}{2}2\frac{b}{2}m\right)\ \left\{\frac{b}{2}m\right\}\left\{\frac{c}{2}2, a\frac{c}{2}2\right\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	25*a*	*Pnmn*	$\{a, b, c\}\left(\frac{c}{2}2\frac{b+c}{2}m:\frac{a}{2}2\right)\ \left\{\frac{b+c}{2}m\right\}\left\{\frac{c}{2}2, \frac{a}{2}2\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$

Table 1 (*cont.*)

	27a	*Pbnb*	$\{a,b,c\}\left(\frac{c}{2}2\frac{b}{2}m_{a/4}:\frac{a}{2}2\right)\ \left\{\frac{b}{2}m_{a/4}\right\}\left\{\frac{c}{2}2,\frac{a}{2}2\right\}$	$a\neq\bar a\ b\neq\bar b\ c\neq\bar c$
	33a	$P4_2$	$\{a,b,c\}\left(\frac{c}{2}4\right)\ \{c\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b$
	36a	$P4_2mc$	$\{a,b,c\}\left(\frac{c}{2}4m\right)\ \{m\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b\ c\neq\bar c$
	37a	$P4_2cm$	$\{a,b,c\}\left(\frac{c}{2}4\frac{c}{2}m\right)\ \left\{\frac{c}{2}m\right\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b\ c\neq\bar c$
	38a	$P4_2nm$	$\{a,b,c\}\left(\frac{c}{2}4\frac{b+c}{2}m_{a/4}\right)\ \left\{\frac{b+c}{2}m_{a/4}\right\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b=c$
	41a	$P\frac{4_2}{m}$	$\{a,b,c\}\left(\frac{c}{2}\dot{4}:m\right)\ \{m\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b\ c\neq\bar c$
	43a	$P42_12$	$\{a,b,c\}\left(4:\frac{a}{2}2_{b/4}\right)\ \{4\}\left\{\frac{a}{2}2_{b/4},c\frac{a}{2}2_{b/4}\right\}$	$a\neq\bar a\ b\neq\bar b$
	44a	$P4_122$	$\{a,b,c\}\left(\frac{c}{4}4:2\right)\ \{2\}\left\{\frac{c}{4}4,a\frac{c}{4}4\right\}$	$a=b\ c\neq\bar c$
	45a	$P4_322$	$\{a,b,c\}\left(-\frac{c}{4}4:2\right)\ \{2\}\left\{-\frac{c}{4}4,-\frac{c}{4}4a\right\}$	$a=b\ c\neq\bar c$
	50a	$P4_22_12_1$	$\{a,b,c\}\left(\frac{c}{2}4:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{a}{2}2_{b/4},c\frac{a}{2}2_{b/4}\right\}$	$a\neq\bar a\ b\neq\bar b$
	52a	$P\bar{4}2_1m$	$\{a,b,c\}\left(\tilde{4}:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{a}{2}2_{b/4}\right\}\{\tilde{4},\tilde{4}c\}$	$a\neq\bar a\ b\neq\bar b$
	84a	$P6_222$	$\{(a,b),c\}\left(\frac{c}{3}6:2\right)\ \left\{\frac{c}{3}6\right\}\{2,2c\}$	$a\neq\bar a\ b\neq\bar b$
	85a	$P6_422$	$\{(a,b),c\}\left(-\frac{c}{3}6:2\right)\ \left\{-\frac{c}{3}6\right\}\{2,2c\}$	$a\neq\bar a\ b\neq\bar b$
	103a	$Pn3m$	$\{a,b,c\}\left(3_{(b-a)/4}\Big/\frac{c}{2}4\frac{b+c}{2}m_{a/4}\right)\ \left\{\frac{b+c}{2}m_{a/4}\right\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}$	$a=b=c\ 3\neq\bar 3$
VII	7s	$P2/m$	$\{a,b,c\}(2:m)\ \{m,cm\}\{2,2a,2b,2ab\}$	
VIII	8s	$B2/m$	$\left\{a,b,\frac{a+c}{2}\right\}(2:m)\ \{m\}\{2,2b\}\left\{\frac{a+b}{2},\frac{a+c}{2}b\right\}$	$a\neq\bar a$
	10s	$C222$	$\left\{a,\frac{a+b}{2},c\right\}(2:2')\ \left\{\frac{a+b}{2}\right\}\{2',22'\}\{2',2'c\}^*$	$a\neq\bar a$
	32s	$P\bar{4}2m$	$\{a,b,c\}(\tilde{4}:2)\ \{2\}\{\tilde{4},\tilde{4}b\}\{\tilde{4},\tilde{4}c\}$	$a=b$
	62a	$P4_2/mnm$	$\{a,b,c\}\left(\frac{c}{2}4\frac{b+c}{2}m_{a/4}:2\right)\ \left\{\frac{b+c}{2}m_{a/4}\right\}\left\{\frac{c}{2}4,a\frac{c}{2}4\right\}\{2,2a\}$	$a=b=c$
IX	9s	$P222$	$\{a,b,c\}(2:2')$ $\{\{c\}\{2,2a,2b,2ab\},\{b\}\{2',2'a,2'c,2'ac\},\{a\}\{22',22'b,22'c,22'bc\}\}$ $\{\{2,2',22'\},\{2a,2'a,22'\},\{2',2b,22'b\},\{2'a,2ab,22'b\},\{2,2'c,22'c\},$ $\{2a,2'ac,22'c\},\{2b,2'c,22'bc\},\{2ab,2'ac,22'bc\}\}$	
X	11s	$I222$	$\left\{a,b,\frac{a+b+c}{2}\right\}(2:2')\ \left\{\frac{a+b+c}{2}\right\}(2,2',22'\}$	$a\neq\bar a\ b\neq\bar b$
	24h	$Fddd$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(2\frac{b+c}{2}m_{a/8}:2'\right)\ \left\{\frac{b+c}{2}m_{a/8}\right\}\{2,2',22'\}$	$a\neq\bar a\ \frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)\ \frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	6a	$I2_12_12_1$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{2}2:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{a+b+c}{2}\right\}\left\{\frac{c}{2}2,\frac{a}{2}2_{b/4},\frac{c}{2}2\frac{a}{2}2_{b/4}\right\}$	$a\neq\bar a\ b\neq\bar b$
XI	12s	$F222$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(2:2')\ \left\{\left\{2,2\frac{a+b}{2}\right\},\left\{2',2'\frac{a+c}{2}\right\},\left\{22',22'\frac{a+b}{2}\frac{a+c}{2}\right\}\right\}$	$a\neq\bar a$
XII	13s	$Pmm2$	$\{a,b,c\}(2m)\ \{c\}\{\{m,am\},\{2m,2bm\}\}$	
	17a	$Pccm$	$\{a,b,c\}\left(2\frac{c}{2}m:2'\right)\ \left\{\frac{c}{2}m\right\}\{\{2,2a\},\{2b,22'b\}\}$	$c\neq\bar c$
XIII	14s	$Cmm2$	$\left\{a,\frac{a+b}{2},c\right\}(2m)\ \left\{\frac{a+b}{2}\right\}\{c\}\{m,2m\}$	$a\neq\bar a$
	15s	$Bmm2$	$\left\{a,b,\frac{a+c}{2}\right\}(2m)\ \left\{\frac{a+c}{2}\right\}\{m\}\{2m,2mb\}$	$a\neq\bar a$
	24s	$P4mm$	$\{a,b,c\}(4m)\ \{c\}\{m\}\{4,4a\}$	$a=b$
	58s	$P6/mmm$	$\{(a,b),c\}(6:m2)\ \{6\}\{2\}\{m,cm\}$	$a\neq\bar a\ b\neq\bar b$
	6h	$Pbm2$	$\{a,b,c\}\left(2\frac{b}{2}m\right)\ \{c\}\left\{\frac{b}{2}m\right\}\{2,2a\}$	$b\neq\bar b$
	11h	$Bma2$	$\left\{a,b,\frac{a+c}{2}\right\}\left(2\frac{c}{2}m\right)\ \left\{\frac{a+c}{2}\right\}\left\{\frac{c}{2}m\right\}\{2,2b\}$	$a\neq\bar a$
	20h	$Cccm$	$\left\{a,\frac{a+b}{2},c\right\}\left(2\frac{c}{2}m:2'\right)\ \left\{\frac{a+b}{2}\right\}\left\{\frac{c}{2}m\right\}\{2,22'\}$	$a\neq\bar a\ c\neq\bar c$
	23h	$Ibam$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(2\frac{c}{2}m:2'\right)\ \left\{\frac{a+b+c}{2}\right\}\left\{\frac{c}{2}m\right\}\{2,22'\}$	$a\neq\bar a\ c\neq\bar c$

Table 1 (*cont.*)

	35*h*	*P4/mcc*	$\{a, b, c\}\left(4\frac{c}{2}m:2\right)\ \{2\}\left\{\frac{c}{2}m\right\}\{4, 4a\}$	$a = b\ c \neq \bar{c}$
	36*h*	*P4/nbm*	$\{a, b, c\}\left(4\frac{b}{2}m_{a/4}:2\right)\ \{4\}\left\{\frac{b}{2}m_{a/4}\right\}\{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	15*a*	*Pnmb*	$\{a, b, c\}\left(\frac{c}{2}2\frac{b+c}{2}m:2\right)\ \{2\}\left\{\frac{b+c}{2}m\right\}\left\{\frac{c}{2}2, a\frac{c}{2}2\right\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	16*a*	*Pbcb*	$\{a, b, c\}\left(\frac{c}{2}2\frac{b}{2}m:2\right)\ \{2\}\left\{\frac{b}{2}m\right\}\left\{\frac{c}{2}2, a\frac{c}{2}2\right\}$	$b \neq \bar{b}\ c \neq \bar{c}$
	23*a*	*Pmca*	$\{a, b, c\}\left(\frac{c}{2}2m:\frac{a}{2}2\right)\ \{m\}\left\{\frac{a}{2}2\right\}\left\{\frac{c}{2}2, b\frac{c}{2}2\right\}$	$a \neq \bar{a}\ c \neq \bar{c}$
	54*a*	*P4/mbm*	$\{a, b, c\}\left(4\frac{b}{2}m_{a/4}:\frac{a}{2}2_{b/4}\right)\ \{4\}\left\{\frac{b}{2}m_{a/4}\right\}\left\{\frac{a}{2}2_{b/4}, c\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	55*a*	*P4/nmm*	$\{a, b, c\}\left(4m:\frac{a}{2}2_{b/4}\right)\ \{4\}\{m\}\left\{\frac{a}{2}2_{b/4}, c\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	60*a*	*P4₂/mmc*	$\{a, b, c\}\left(\frac{c}{2}4m:2\right)\ \{2\}\{m\}\left\{\frac{c}{2}4, a\frac{c}{2}4\right\}$	$a = b\ c \neq \bar{c}$
	61*a*	*P4₂/mcm*	$\{a, b, c\}\left(\frac{c}{2}4\frac{c}{2}m:2\right)\ \{2\}\left\{\frac{c}{2}m\right\}\left\{\frac{c}{2}4, a\frac{c}{2}4\right\}$	$a = b\ c \neq \bar{c}$
XIV	17*s*	*Fmm2*	$\left\{a, \frac{a+b}{2}, \frac{a+c}{2}\right\}(2m)\ \left\{\frac{a+c}{2}, \frac{a+c}{2}\frac{a+b}{2}\right\}\{m, 2m\}\left\{\frac{a+c}{2}m, \frac{a+c}{2}\frac{a+b}{2}2m\right\}$	$a \neq \bar{a}$
	22*h*	*Ccca*	$\left\{a, \frac{a+b}{2}, c\right\}\left(2\frac{b+c}{2}m:2'\right)$	
			$\{2', 22'\}\left\{\frac{b+c}{2}m, \frac{a+b}{2}\frac{b+c}{2}m\right\}\left\{2'\frac{b+c}{2}m, 22'\frac{a+b}{2}\frac{b+c}{2}m\right\}$	$a \neq \bar{a}\ c \neq \bar{c}$
	20*a*	*Imcm*	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(\frac{c}{2}2m:\frac{a}{2}2_{b/4}\right)$	
			$\left\{\frac{c}{2}2, \frac{c}{2}2m\right\}\left\{\frac{a}{2}2_{b/4}, \frac{a+b+c}{2}\frac{a}{2}2_{b/4}\right\}\left\{\frac{c}{2}2\frac{a}{2}2_{b/4}, \frac{c}{2}2m, \frac{a+b+c}{2}\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
XV	18*s*	*Pmmm*	$\{a, b, c\}(2m:2')\ \{\{m, ma\}, \{2m, 2mb\}, \{22'm, 22'mc\}\}$	
XVI	19*s*	*Cmmm*	$\left\{a, \frac{a+b}{2}, c\right\}(2m:2')\ \left\{\frac{a+b}{2}\right\}\{m, 2m\}\{22'm, 22'mc\}$	$a \neq \bar{a}$
	36*s*	*P4/mmm*	$\{a, b, c\}(4m:2)\ \{m\}\{4, 4a\}\{4a2, 4a2c\}$	$a = b$
	14*a*	*Pmcm*	$\{a, b, c\}\left(\frac{c}{2}2m:2\right)\ \{m\}\{2, 2a\}\left\{2\frac{c}{2}2b, 2\frac{c}{2}2ab\right\}$	$c \neq \bar{c}$
XVII	20*s*	*Immm*	$\left\{a, b, \frac{a+b+c}{2}\right\}(2m:2')\ \left\{\frac{a+b+c}{2}\right\}\{m, 2m, 22'm\}$	$a \neq \bar{a}\ b \neq \bar{b}$
XVIII	21*s*	*Fmmm*	$\left\{a, \frac{a+b}{2}, \frac{a+c}{2}\right\}(2m:2')\ \left\{\left(\frac{a+b}{2}, m\right), \left(\frac{a+c}{2}, 2m\right), \left(\frac{a+b}{2}\frac{a+b}{2}, 22'm\right)\right\}^{\dagger}$	$a \neq \bar{a}$
XIX	23*s*	*I4*	$\left\{a, b, \frac{a+b+c}{2}\right\}(4)\ \{4\}\left\{\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	40*s*	*P3m1*	$\{(a, b), c\}(3m)\ \{c\}\{m\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	41*s*	*P31m*	$\{(a, b), c\}(m3)\ \{c\}\{m\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	42*s*	*R3m*	$\{a, b, c\}(m3)\ \{a\}\{m\}$	$a = b = c\ 3 \neq \bar{3}$
	49*s*	*P6*	$\{(a, b), c\}(6)\ \{6\}\{c\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	63*s*	*Im3*	$\left\{a, b, \frac{a+b+c}{2}\right\}(3/2m)\ \left\{\frac{a+b+c}{2}\right\}\{m\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}\ 2 \neq \bar{2}$
	65*s*	*P4̄3m*	$\{a, b, c\}(3/\bar{4})\ \{\bar{4}\}\{a\}$	$a = b = c\ 3 \neq \bar{3}$
	66*s*	*I4̄3m*	$\left\{a, b, \frac{a+b+c}{2}\right\}(3/\bar{4})\ \{\bar{4}\}\left\{\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	69*s*	*I432*	$\left\{a, b, \frac{a+b+c}{2}\right\}(3/4)\ \{4\}\left\{\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	73*s*	*Fm3m*	$\left\{a, \frac{a+b}{2}, \frac{a+c}{2}\right\}(3/4m)\ \{4\}\{m\}$	$a \neq \bar{a}\ \frac{a+b}{2} \neq \left(\overline{\frac{a+b}{2}}\right) \frac{a+c}{2} \neq \left(\overline{\frac{a+c}{2}}\right) 3 \neq \bar{3}$
	27*h*	*P4nc*	$\{a, b, c\}\left(4\frac{b+c}{2}m_{a/4}\right)\ \{4\}\left\{\frac{b+c}{2}m_{a/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	42*h*	*P6̄2c*	$\{(a, b), c\}(3:m_{c/4}2)\ \{2\}\{m_{c/4}\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$
	43*h*	*P6̄c2*	$\{(a, b), c\}(2m_{c/4}:3)\ \{2\}\{m_{c/4}\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$
	44*h*	*P6cc*	$\{(a, b), c\}\left(6\frac{c}{2}m\right)\ \{6\}\left\{\frac{c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	45*h*	*P3̄c1*	$\{(a, b), c\}\left(3\frac{c}{2}m:2\right)\ \{2\}\left\{\frac{c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$
	46*h*	*P3̄1c*	$\{(a, b), c\}\left(2:\frac{c}{2}m3\right)\ \{2\}\left\{\frac{c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$
	47*h*	*R3̄c*	$\{a, b, c\}\left(2:\frac{a+b+c}{2}m3\right)\ \{2\}\left\{\frac{a+b+c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$

Table 1 (*cont.*)

	53*h*	*Pn*3*n*	$\{a,b,c\}\left(3/4\frac{b+c}{2}m_{a/4}\right)\ \{4\}\left\{\frac{b+c}{2}m_{a/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}\;3\neq\bar{3}$
	54*h*	*Fm*3*c*	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(3/4\frac{c}{2}m\right)\ \{4\}\left\{\frac{c}{2}m\right\}$	$a\neq\bar{a}\;\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)\;\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$ $3\neq\bar{3}$
	12*a*	$Pbn2_1$	$\{a,b,c\}\left(\frac{c}{2}2\frac{b}{2}m_{a/4}\right)\ \left\{\frac{c}{2}2\right\}\left\{\frac{b}{2}m_{a/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	32*a*	$I4_1$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{4}4\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{a+b+c}{2}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	34*a*	$I4_1md$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{4}4\frac{b+c}{2}m\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{b+c}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	35*a*	$I4_1cd$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{4}4\frac{b}{2}m\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{b}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	39*a*	$P4_2bc$	$\{a,b,c\}\left(\frac{c}{2}4\frac{b}{2}m_{a/4}\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b}{2}m_{a/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c=\bar{c}$
	40*a*	$I4_1/a$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{4}4\frac{a}{2}m\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{a}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	48*a*	$P4_12_12$	$\{a,b,c\}\left(\frac{c}{4}4:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	49*a*	$P4_32_12$	$\{a,b,c\}\left(-\frac{c}{4}4:\frac{a}{2}2_{b/4}\right)\ \left\{-\frac{c}{4}4\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	51*a*	$I\bar{4}2d$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\tilde{4}:\frac{a}{2}2_{c/8}\right)\ \{\tilde{4}\}\left\{\frac{a}{2}2_{c/8}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	53*a*	$P\bar{4}2_1c$	$\{a,b,c\}\left(\tilde{4}:\frac{a}{2}2_{(b+c)/4}\right)\ \{\tilde{4}\}\left\{\frac{a}{2}2_{(b+c)/4}\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	76*a*	$P6_2$	$\{(a,b),c\}\left(\frac{c}{3}6\right)\ \left\{\frac{c}{3}6\right\}\{c\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	77*a*	$P6_4$	$\{(a,b),c\}\left(-\frac{c}{3}6\right)\ \left\{-\frac{c}{3}6\right\}\{c\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	79*a*	$P6_3mc$	$\{(a,b),c\}\left(\frac{c}{2}6m\right)\ \left\{\frac{c}{2}6\right\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	80*a*	$P6_3cm$	$\{(a,b),c\}\left(\frac{c}{2}6\frac{c}{2}m\right)\ \left\{\frac{c}{2}6\right\}\left\{\frac{c}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	81*a*	$P6_3/m$	$\{(a,b),c\}\left(\frac{c}{2}6:m\right)\ \left\{\frac{}{2}6\right\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	82*a*	$P6_122$	$\{(a,b),c\}\left(\frac{c}{6}6:2\right)\ \left\{\frac{c}{6}6\right\}\{2\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	83*a*	$P6_522$	$\{(a,b),c\}\left(-\frac{c}{6}6:2\right)\ \left\{-\frac{c}{6}6\right\}\{2\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	86*a*	$P6_322$	$\{(a,b),c\}\left(\frac{c}{2}6:2\right)\ \left\{\frac{c}{2}6\right\}\{2\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}$
	96*a*	$I4_132$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(3_{(b-a)/8}\Big/\frac{c}{4}4\right)\ \left\{\frac{a+b+c}{2}\right\}\left\{\frac{c}{4}4\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;3\neq\bar{3}$
	99*a*	*Ia*3*d*	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(3_{(b-a)/8}\Big/\frac{c}{4}4\frac{b}{2}m\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{b}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}\;3\neq\bar{3}\;\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	100*a*	*Fd*3*m*	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(3_{(b-a)/8}\Big/\frac{c}{4}4\frac{b+c}{4}m_{a/8}\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{b+c}{4}m_{a/8}\right\}$	$a\neq\bar{a}\;\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)3\neq\bar{3}$
	101*a*	*Fd*3*c*	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(3_{(b-a)/8}\Big/\frac{c}{4}4\frac{b-c}{4}m_{a/8}\right)\ \left\{\frac{c}{4}4\right\}\left\{\frac{b-c}{4}m_{a/8}\right\}$	$a\neq\bar{a}\;\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)3\neq\bar{3}$
	102*a*	*Pm*3*n*	$\{a,b,c\}\left(3_{(b-a)/4}\Big/\frac{c}{2}4m\right)\ \left\{\frac{c}{2}4\right\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}\;c\neq\bar{c}\;3\neq\bar{3}$
XX	25*s*	*I*4*mm*	$\left\{a,b,\frac{a+b+c}{2}\right\}(4m)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	29*s*	*I*4/*m*	$\left\{a,b,\frac{a+b+c}{2}\right\}(4:m)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	31*s*	*I*422	$\left\{a,b,\frac{a+b+c}{2}\right\}(4:2)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{2\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	34*s*	$I\bar{4}2m$	$\left\{a,b,\frac{a+b+c}{2}\right\}(\tilde{4}:2)\ \left\{\frac{a+b+c}{2}\right\}\{\tilde{4}\}\{2\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	50*s*	*P*6*mm*	$\{(a,b),c\}(6m)\ \{6\}\{c\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}$
	72*s*	*Im*3*m*	$\left\{a,b,\frac{a+b+c}{2}\right\}(3/4m)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{m\}$	$a\neq\bar{a}\;b\neq\bar{b}\;3\neq\bar{3}$
	12*h*	*Bbm*2	$\left\{a,b,\frac{a+c}{2}\right\}\left(2\frac{b}{2}m\right)\ \left\{\frac{a+c}{2}\right\}\{2\}\left\{\frac{b}{2}m\right\}$	$a\neq\bar{a}\;b\neq\bar{b}$

Table 1 (*cont.*)

	13*h*	*Bba*2	$\left\{a, b, \frac{a+c}{2}\right\}\left(2\frac{b+c}{2}m\right)\ \left\{\frac{a+c}{2}\right\}\{2\}\left\{\frac{b+c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	14*h*	*Ibm*2	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(2\frac{b}{2}m\right)\ \left\{\frac{a+b+c}{2}\right\}\{2\}\left\{\frac{b}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	26*h*	*P*4*bm*	$\{a, b, c\}\left(4\frac{b}{2}m_{a/4}\right)\ \{c\}\{4\}\left\{\frac{b}{2}m_{a/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	28*h*	*I*4*cm*	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(4\frac{c}{2}m\right)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\left\{\frac{c}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	37*h*	*P*4/*nnc*	$\{a, b, c\}\left(4\frac{b+c}{2}m_{a/4}:2\right)\ \{4\}\left\{\frac{b+c}{2}m_{a/4}\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	48*h*	*P*6/*mcc*	$\{(a, b), c\}(6:m_{c/4}2)\ \{6\}\{2\}\{m_{c/4}\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	13*a*	*Cmc*2_1	$\left\{a, \frac{a+b}{2}, c\right\}\left(\frac{c}{2}2m\right)\ \left\{\frac{a+b}{2}\right\}\left\{\frac{c}{2}2\right\}\{m\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	17*a*	*Pbnn*	$\{a, b, c\}\left(\frac{c}{2}2\frac{b}{2}m_{a/4}:2\right\}\ \left\{\frac{c}{2}2\right\}\left\{\frac{b}{2}m_{a/2}\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	26*a*	*Pbcn*	$\{a, b, c\}\left(\frac{c}{2}2\frac{b}{2}m:\frac{a}{2}2\right)\ \left\{\frac{c}{2}2\right\}\left\{\frac{b}{2}m\right\}\left\{\frac{a}{2}2\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	28*a*	*Pmcn*	$\{a, b, c\}\left(\frac{c}{2}2m:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}2\right\}\{m\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	46*a*	*I*$4_1$22	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(\frac{c}{4}4:2\right)\ \left\{\frac{a+b+c}{2}\right\}\left\{\frac{c}{4}4\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	56*a*	*P*4/*mnc*	$\{a, b, c\}\left(4\frac{b+c}{2}m_{a/4}:\frac{a}{2}2_{b/4}\right)\ \{4\}\left\{\frac{b+c}{2}m_{a/4}\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	57*a*	*P*4/*nnc*	$\{a, b, c\}\left(4\frac{c}{2}m:\frac{a}{2}2_{b/4}\right)\ \{4\}\left\{\frac{c}{2}m\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	58*a*	*I*4_1/*acd*	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(\frac{c}{2}4\frac{b}{2}m:2\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b}{2}m\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ \frac{a+b+c}{2} \neq \left(\overline{\frac{a+b+c}{2}}\right)$
	59*a*	*I*4_1/*amd*	$\left\{a, b, \frac{a+b+c}{2}\right\}\left(\frac{c}{2}4\frac{b+c}{2}m:2\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b+c}{2}m\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ \frac{a+b+c}{2} \neq \left(\overline{\frac{a+b+c}{2}}\right)$
	63*a*	*P*4_2/*nbc*	$\{a, b, c\}\left(\frac{c}{2}4\frac{b}{2}m_{a/4}:2\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b}{2}m_{a/4}\right\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	64*a*	*P*4_2/*mnm*	$\{a, b, c\}\left(\frac{c}{2}4\frac{b+c}{2}m_{a/4}:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b+c}{2}m_{a/4}\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	65*a*	*P*4_2/*ncm*	$\{a, b, c\}\left(\frac{c}{2}4\frac{c}{2}m:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{c}{2}m\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	66*a*	*P*4_2/*mbc*	$\{a, b, c\}\left(\frac{c}{2}4\frac{b}{2}m_{a/4}:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}4\right\}\left\{\frac{b}{2}m_{a/4}\right\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	67*a*	*P*4_2/*nmc*	$\{a, b, c\}\left(\frac{c}{2}4m:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{c}{2}4\right\}\{m\}\left\{\frac{a}{2}2_{b/4}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	87*a*	*P*6_3/*mmc*	$\{(a, b), c\}\left(\frac{c}{2}6:m_{c/4}2\right)\ \left\{\frac{c}{2}6\right\}\{m_{c/4}\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
	88*a*	*P*6_3/*mcm*	$\{(a, b), c\}\left(\frac{c}{2}6:m2\right)\ \left\{\frac{c}{2}6\right\}\{m\}\{2\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c \neq \bar{c}$
XXI	27*s*	*I*$\bar{4}$	$\left\{a, b, \frac{a+b+c}{2}\right\}(\tilde{4})\ \left\{\tilde{4}, \tilde{4}\frac{a+b+c}{2}\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	43*s*	*P*$\bar{6}$	$\{(a, b), c\}(3:m)\ \{m, cm\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	44*s*	*P*321	$\{(a, b), c\}(3:2)\ \{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	45*s*	*P*312	$\{(a, b), c\}(2:3)\ \{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}\ 3 \neq \bar{3}$
	46*s*	*R*32	$\{a, b, c\}(2:3)\ \{2, 2a\}$	$a = b = c\ 3 \neq \bar{3}$
	51*s*	*P*$\bar{3}$	$\{(a, b), c\}(\tilde{6})\ \{\tilde{6}, \tilde{6}c\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	52*s*	*R*$\bar{3}$	$\{a, b, c\}(\tilde{6})\ \{\tilde{6}, \tilde{6}a\}$	$a = b = c$
	62*s*	*Pm*3	$\{a, b, c\}(3/2m)\ \{m, ma\}$	$a = b = c\ 3 \neq \bar{3}\ 2 \neq \bar{2}$
	68*s*	*P*432	$\{a, b, c\}(3/4)\ \{4, 4a\}$	$a = b = c\ 3 \neq \bar{3}$
	2*h*	*Bb*	$\left\{a, b, \frac{a+c}{2}\right\}\left(\frac{b}{2}m\right)\ \left\{\frac{b}{2}m, \frac{a+c}{2}\frac{b}{2}m\right\}$	$a \neq \bar{a}\ b \neq \bar{b}$
	16*h*	*Fdd*2	$\left\{a, \frac{a+b}{2}, \frac{a+c}{2}\right\}\left(2\frac{b+c}{4}m_{a/8}\right)\ \left\{\frac{b+c}{4}m_{a/8}, 2\frac{b+c}{4}m_{a/8}\right\}$	$a \neq \bar{a}\ \frac{a+b}{2} \neq \left(\overline{\frac{a+b}{2}}\right)\ \frac{a+c}{2} \neq \left(\overline{\frac{a+c}{2}}\right)$
	49*h*	*Pn*3	$\{a, b, c\}\left(3/2\frac{b+c}{2}m_{a/4}\right)\ \left\{\frac{b+c}{2}m_{a/4}, \frac{b+c}{2}m_{a/4}a\right\}$	$b \neq \bar{b}\ c \neq \bar{c}\ 3 \neq \bar{3}$
	30*a*	*P*4_1	$\{a, b, c\}\left(\frac{c}{4}4\right)\ \left\{\frac{c}{4}4, a\frac{c}{4}4\right\}$	$a = b\ c \neq \bar{c}$
	31*a*	*P*4_3	$\{a, b, c\}\left(-\frac{c}{4}4\right)\ \left\{-\frac{c}{4}4, -\frac{c}{4}4a\right\}$	$a = b\ c \neq \bar{c}$
	70*a*	*P*$3_1$21	$\{(a, b), c\}\left(\frac{c}{3}3:2\right)\ \{2, 2c\}$	$a \neq \bar{a}\ b \neq \bar{b}\ c = \frac{c}{3}3$

Table 1 (*cont.*)

	71*a*	$P3_221$	$\{(a,b),c\}\left(-\frac{c}{3}3:2\right)\ \{2,2c\}$	$a\neq\bar a\, b\neq\bar b\, c=-\frac{c}{3}3$
	72*a*	$P3_112$	$\{(a,b),c\}\left(2:\frac{c}{3}3\right)\ \{2,2c\}$	$a\neq\bar a\, b\neq\bar b\, c=\frac{c}{3}3$
	73*a*	$P3_212$	$\{(a,b),c\}\left(2:-\frac{c}{3}3\right)\ \{2,2c\}$	$a\neq\bar a\, b\neq\bar b\, c=-\frac{c}{3}3$
	92*a*	$Ia3$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(3_{(a-b)/8}\ \frac{c}{2}2\frac{b}{2}m\right)\ \left\{\frac{b}{2}m,\frac{b}{2}m\frac{a+b+c}{2}\right\}$	$a\neq\bar a\, b\neq\bar b\frac{c}{2}2\neq\left(\overline{\frac{c}{2}2}\right)3\neq\bar 3$
	98*a*	$P4_232$	$\{a,b,c\}\left(3_{(b-a)/4}\Big/\frac{c}{2}4\right)\ \left\{\frac{c}{2}4, a\frac{c}{2}4\right\}$	$a=b=c\,3\neq\bar 3$
XXII	28*s*	$P4/m$	$\{a,b,c\}(4:m)\ \{4,4a\}\{m,cm\}$	$a=b$
	30*s*	$P422$	$\{a,b,c\}(4:2)\ \{4,4a\}\{2,2c\}$	$a=b$
	33*s*	$P\bar 4m2$	$\{a,b,c\}(\bar 4m)\ \{m,am\}\{\bar 4,\bar 4c\}$	$a=b$
	18*h*	$Pncb$	$\{a,b,c\}\left(2\frac{b+c}{2}m:2'\right)\ \{2,22'\}\left\{\frac{b+c}{2}m,\frac{b+c}{2}ma\right\}$	$b\neq\bar b\, c\neq\bar c$
	9*a*	$Pmc2_1$	$\{a,b,c\}\left(\frac{c}{2}2m\right)\ \{m,ma\}\left\{\frac{c}{2}2,b\frac{c}{2}2\right\}$	$c\neq\bar c$
	22*a*	$Pcma$	$\{a,b,c\}\left(\frac{c}{2}2\frac{c}{2}m:\frac{a}{2}2\right)\ \left\{\frac{c}{2}m,\frac{c}{2}m\frac{c}{2}2\frac{a}{2}2\right\}\left\{\frac{c}{2}m\frac{c}{2}2,b\frac{c}{2}m\frac{c}{2}2\right\}$	$a\neq\bar a\, c\neq\bar c$
	24*a*	$Pmnm$	$\{a,b,c\}\left(\frac{c}{2}2m_{a/4}:\frac{a}{2}2\right)\ \left\{m_{a/4},\frac{c}{2}2\frac{a}{2}2m_{a/4}\right\}\left\{\frac{c}{2}2m_{a/4},b\frac{c}{2}2m_{a/4}\right\}$	$a\neq\bar a\, c\neq\bar c$
	47*a*	$P4_222$	$\{a,b,c\}\left(\frac{c}{2}4:2\right)\ \left\{\frac{c}{2}4,a\frac{c}{2}4\right\}\{2,2c\}$	$a\neq\bar a\, b\neq\bar b$
XXIII	37*s*	$I4/mmm$	$\left\{a,b,\frac{a+b+c}{2}\right\}(4m:2)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{2\}\{m\}$	$a\neq\bar a\, b\neq\bar b$
	38*h*	$I4/mcm$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(4\frac{c}{2}m:2\right)\ \left\{\frac{a+b+c}{2}\right\}\{4\}\{2\}\left\{\frac{c}{2}m\right\}$	$a\neq\bar a\, b\neq\bar b$
	18*a*	$Cmcm$	$\left\{a,\frac{a+b}{2},c\right\}\left(\frac{c}{2}2m:2\right)\ \left\{\frac{a+b}{2}\right\}\left\{\frac{c}{2}2\right\}\{m\}\{2\}$	$a\neq\bar a\, b\neq\bar b$
	19*a*	$Cmca$	$\left\{a,\frac{a+b}{2},c\right\}\left(\frac{c}{2}2\frac{b}{2}m:2\right)\ \left\{\frac{a+b}{2}\right\}\left\{\frac{c}{2}2\right\}\left\{\frac{b}{2}m\right\}\{2\}$	$a\neq\bar a\, b\neq\bar b$
XXIV	38*s*	$P3$	$\{(a,b),c\}(3)\ \{c\}$	$a\neq\bar a\, b\neq\bar b\,3\neq\bar 3$
	39*s*	$R3$	$\{a,b,c\}(3)\ \{a\}$	$a=b=c\,3\neq\bar 3$
	59*s*	$P23$	$\{a,b,c\}(3/2)\ \{a\}$	$a=b=c\,3\neq\bar 3\,2\neq\bar 2$
	60*s*	$I23$	$\left\{a,b,\frac{a+b+c}{2}\right\}(3/2)\ \left\{\frac{a+b+c}{2}\right\}$	$a\neq\bar a\, b\neq\bar b\, c\neq\bar c\,3\neq\bar 3\,2\neq\bar 2$
	64*s*	$Fm3$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(3/2m)\ \{m\}$	$a\neq\bar a\,3\neq\bar 3\,2\neq\bar 2\,\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	67*s*	$F\bar 43m$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(3/\bar 4)\ \{\bar 4\}$	$a\neq\bar a\,3\neq\bar 3\,\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	70*s*	$F432$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(3/4)\ \{4\}$	$a\neq\bar a\,3\neq\bar 3\,\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	39*h*	$P3c1$	$\{(a,b),c\}\left(3\frac{c}{2}m\right)\ \left\{\frac{c}{2}m\right\}$	$a\neq\bar a\, b\neq\bar b\, c\neq\bar c\,3\neq\bar 3$
	40*h*	$P31c$	$\{(a,b),c\}\left(\frac{c}{2}m3\right)\ \left\{\frac{c}{2}m\right\}$	$a\neq\bar a\, b\neq\bar b\, c\neq\bar c\,3\neq\bar 3$
	41*h*	$R3c$	$\{a,b,c\}\left(\frac{a+b+c}{2}m3\right)\ \left\{\frac{a+b+c}{2}m\right\}$	$a\neq\bar a\, b\neq\bar b\, c\neq\bar c\,3\neq\bar 3$
	50*h*	$Fd3$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(3/2\frac{b+c}{4}m_{a/8}\right)\ \left\{\frac{b+c}{4}m_{a/8}\right\}$	$a\neq\bar a\,\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)2\neq\bar 2\,3\neq\bar 3$
	51*h*	$P\bar 43n$	$\{a,b,c\}(3/\bar 4_{(2a+c)/4})\ \{\bar 4_{(2a+c)/4}\}$	$a\neq\bar a\, b\neq\bar b\, c\neq\bar c\,3\neq\bar 3$
	52*h*	$F\bar 43c$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(3/\bar 4_{c/4})\ \{\bar 4_{c/4}\}$	$a\neq\bar a\,3\neq\bar 3\,\frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	68*a*	$P3_1$	$\{(a,b),c\}\left(\frac{c}{3}3\right)\ \{c\}$	$a\neq\bar a\, b\neq\bar b\, c=\frac{c}{3}3$
	69*a*	$P3_2$	$\{(a,b),c\}\left(-\frac{c}{3}3\right)\ \{c\}$	$a\neq\bar a\, b\neq\bar b\, c=-\frac{c}{3}3$

Table 1 (*cont.*)

	74*a*	$P6_1$	$\{(a,b),c\}\left(\frac{c}{6}6\right)\ \left\{\frac{c}{6}6\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}$
	75*a*	$P6_5$	$\{(a,b),c\}\left(-\frac{c}{6}6\right)\ \left\{-\frac{c}{6}6\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}$
	78*a*	$P6_3$	$\{(a,b),c\}\left(\frac{c}{2}6\right)\ \left\{\frac{c}{2}6\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}$
	90*a*	$I2_13$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(3_{(a-b)/8}\big/\frac{c}{2}2\right)\ \left\{\frac{a+b+c}{2}\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; 3\neq\bar{3}\; \frac{c}{2}2\neq\left(\overline{\frac{c}{2}2}\right)$
	91*a*	$Pa3$	$\{a,b,c\}\left(3_{(a-b)/8}\big/\frac{c}{2}2\frac{b}{2}m\right)\ \left\{\frac{b}{2}m\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}\; 3\neq\bar{3}$ $\frac{c}{2}2\neq\left(\overline{\frac{c}{2}2}\right)$
	93*a*	$I\bar{4}3d$	$\left\{a,b,\frac{a+b+c}{2}\right\}(3_{(a-b+2c)/8}/\bar{4})\ \{\bar{4}\}$	$a\neq\bar{a}\; b\neq\bar{b}\; 3\neq\bar{3}$ $\frac{a+b+c}{2}\neq\left(\overline{\frac{a+b+c}{2}}\right)$
	94*a*	$P4_132$	$\{a,b,c\}\left(3_{(b-a)/8}\big/\frac{c}{2}4\right)\ \left\{\frac{c}{4}4\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}\; 3\neq\bar{3}$
	95*a*	$P4_332$	$\{a,b,c\}\left(3_{(a-b)/8}\big/-\frac{c}{4}4\right)\ \left\{-\frac{c}{4}4\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}\; 3\neq\bar{3}$
	97*a*	$F4_132$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}\left(3_{(b-a)/8}\big/\frac{c}{4}4\right)\ \left\{\frac{c}{4}4\right\}$	$a\neq\bar{a}\; 3\neq\bar{3}\; \frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
XXV	61*s*	$F23$	$\left\{a,\frac{a+b}{2},\frac{a+c}{2}\right\}(3/2)$	$a\neq\bar{a}\; 3\neq\bar{3}\; 2\neq\bar{2}\; \frac{a+b}{2}\neq\left(\overline{\frac{a+b}{2}}\right)$ $\frac{a+c}{2}\neq\left(\overline{\frac{a+c}{2}}\right)$
	89*a*	$P2_13$	$\{a,b,c\}\left(3_{(a-b)/8}\big/\frac{c}{2}2\right)$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}\; 3\neq\bar{3}$ $\frac{c}{2}2\neq\left(\overline{\frac{c}{2}2}\right)$
XXVI	3*h*	$P2/b$	$\{a,b,c\}\left(2:\frac{b}{2}m\right)\ \{2,2a\}\{2c,2ac\}\left\{\left\{\frac{b}{2}m,c\frac{b}{2}m\right\},\left\{a\frac{b}{2}m,ac\frac{b}{2}m\right\}\right\}$	$b\neq\bar{b}$
XXVII	5*h*	$Pcc2$	$\{a,b,c\}\left(2\frac{c}{2}m\right)\ \left\{\left\{\frac{c}{2}m,\frac{c}{2}ma\right\},\left\{\frac{c}{2}m2,\frac{c}{2}m2b\right\}\right\}$	$c\neq\bar{c}$
	4*a*	$P222_1$	$\{a,b,c\}\left(\frac{c}{2}2:2\right)\ \left\{\{2,2a\},\left\{2\frac{c}{2}2,2\frac{c}{2}2b\right\}\right\}$	$c\neq\bar{c}$
XXVIII	8*h*	$Pnn2$	$\{a,b,c\}\left(2\frac{b+c}{2}m_{a/4}\right)\ \{2,2a\}\left\{\frac{b+c}{2}m_{a/4},\frac{b+c}{2}m_{a/4}a,\frac{b+c}{2}m_{a/4}2,\frac{b+c}{2}m_{a/4}2a\right\}$	$a=b=c$
XXIX	19*h*	$Pnnn$	$\{a,b,c\}\left(2\frac{b+c}{2}m_{a/4}:2'\right)\ \left\{\frac{b+c}{2}m_{a/4},\frac{b+c}{2}m_{a/4}a\right\}\{\{2,2a\},\{2',2'a\},\{22',22'a\}\}$	$a=b=c$
XXX	21*h*	$Cmma$	$\left\{a,\frac{a+b}{2},c\right\}\left(2\frac{b}{2}m:2'\right)$ $\left(22'\frac{b}{2}m,2\frac{a+b}{2}\frac{b}{2}m\right)\left(22'\frac{a+b}{2}\frac{b}{2}m,2\frac{b}{2}m\right)\ \left\{\left(\frac{b}{2}m,\frac{a+b}{2}\frac{b}{2}m\right),\left(\frac{b}{2}mc,\frac{a+b}{2}\frac{b}{2}mc\right)\right\}$	$a\neq\bar{a}$
XXXI	1*a*	$P2_1$	$\{a,b,c\}\left(\frac{c}{2}2\right)\ \left\{\frac{c}{2}2,\frac{c}{2}2a,\frac{c}{2}2b,\frac{c}{2}2ab\right\}$	$c\neq\bar{c}$
XXXII	8*a*	$P2_12_12_1$	$\{a,b,c\}\left(\frac{c}{2}2:\frac{a}{2}2_{b/4}\right)\ \left\{\frac{a}{2}2_{b/4},\frac{c}{2}2,\frac{a}{2}2_{b/4}\frac{c}{2}2\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}$
XXXIII	21*a*	$Ibca$	$\left\{a,b,\frac{a+b+c}{2}\right\}\left(\frac{c}{2}2\frac{b}{2}m:\frac{a}{2}2_{b/4}\right)$ $\left\{\frac{a+b+c}{2}\right\}\left(\frac{c}{2}2,\frac{a}{2}2_{b/4},\frac{c}{2}2\frac{a}{2}2_{b/4}\right)$ $\left\{\left(\frac{b}{2}m,\frac{b}{2}m\frac{c}{2}2,\frac{b}{2}m\frac{c}{2}2\frac{a}{2}2_{b/4}\right),\left(\frac{b}{2}m\frac{a+b+c}{2},\frac{b}{2}m\frac{c}{2}2\frac{a+b+c}{2},\frac{b}{2}m\frac{c}{2}2\frac{a}{2}2_{b/4}\frac{a+b+c}{2}\right)\right\}$	$a\neq\bar{a}\; b\neq\bar{b}$
XXXIV	29*a*	$Pbca$	$\{a,b,c\}\left(\frac{c}{2}2\frac{b}{2}m:\frac{a}{2}2_{b/4}\right)$ $\left\{\frac{b}{2}m\frac{a}{2}2_{b/4}\right\}\left\{\left(\frac{c}{2}2,\frac{b}{2}m\right),\left(\frac{a}{2}2_{b/4},\frac{b}{2}m\frac{c}{2}2\right),\left(\frac{c}{2}2\frac{a}{2}2_{b/4},\frac{c}{2}2\frac{b}{2}m\frac{a}{2}2_{b/4}\right)\right\}$	$a\neq\bar{a}\; b\neq\bar{b}\; c\neq\bar{c}$

* If for different transformations the same symbol is used in the antisymmetric characteristic $AC(G)$, the different transformations are indicated by ′.
† All the terms in the ( ) brackets of AC are commutated simultaneously.

$AC^m(G_i)$ consists is the type of the antisymmetric characteristic $AC^m(G_i)$.

Theorem 1: Two simple or multiple antisymmetry M^m-type G_i^m and G_j^m groups $(i, j \in N)$ that have the same type of antisymmetric characteristics AC^m generate the same number of multiple antisymmetry M^{m+1}-type groups. The same number of groups with the same types of antisymmetric characteristics AC^{m+1} occur between the multiple antisymmetry M^{m+1}-type groups derived from the G_i^m and G_j^m groups. All the multiple antisymmetry M^{m+1}-type groups which are obtained from the G_i^m group by the general method of Shubnikov-Zamorzaev are known, so by transition from m to $m+1$ it is possible to determine all the multiple antisymmetry M^{m+1}-type groups derived from the G_j^m group $(1 \leq m \leq q-1)$.

The application of theorem 1 allows the classification of simple and multiple antisymmetry groups, generated in one family, with common generating symmetry group G into equivalency classes which consist of the groups with the same type of antisymmetric characteristics, *i.e.* the partial cataloguing of simple and multiple antisymmetry M^m-type groups $(2 \leq m \leq q)$.

In the partial catalogue of simple and multiple antisymmetric M^m-type space groups $(m \leq 2)$, in each of the I-XXXIV classes, the list of groups (*i.e.* the list of corresponding antisymmetric characteristics) which are representatives of equivalency classes, formed of groups with different antisymmetric characteristic types, is stated.

By application of the partial cataloguing algorithm (Jablan, 1986*b*), it is possible to compute the numbers $N_m(G)$ and N_m and to catalogue completely the M^m-type groups.

In the partial catalogue* constructed according to the method of simple and multiple M^m-type group partial cataloguing (Jablan, 1986*b*), arranged in the I-XXXIV classes, the data are given in the following order: the number m, the antisymmetric characteristic AC^{m-1} of the generating antisymmetry group $(m \leq 2)$, the antisymmetric characteristic of the derived groups of ${}^mM$ or M^m type, of which the partial catalogue consists, the symbol (${}^mM$ or M^m) of the type of group and the antisymmetric characteristic type AC^m, for each of the M^m-type groups.

When the quotation of groups of the M^m type $(m \geq 2)$ is not necessary, *i.e.* when the antisymmetry group is unique in the corresponding equivalency class, determined by the relation of equivalence of the AC^{m-1} antisymmetric characteristic types, or the transformation $f_0: e^+ \leftrightarrow E$, where e^+ is the product of the $e_1, e_2, \ldots, e_{m-1}$ antiidentities, does not exist (Jablan, 1986*b*), a + sign is given beside the antisymmetric characteristic of the generating group of simple or multiple antisymmetry of M^{m-1} type $(m \geq 2)$.

From these results, in each of the I-XXXIV classes the numbers $N_m(G)$ and N_m are computed. The $N_m(G)$ numbers, which correspond to the I-XXXIV classes, and the N_m numbers $(1 \leq m \leq 6)$, are given in Table 2. To shorten Table 2, within the antisymmetric characteristics only antiidentities and their products which belong to corresponding antigenerators are given.

The method of partial catalogue usage, the computation of the $N_m(G)$ number and the complete cataloguing process are now illustrated with the $Bm(6s)$ group, a representative of class VI:

Bm $6s$	$\left\{a, b, \frac{a+c}{2}\right\}(m)$		$\{m\}\left\{\frac{a+c}{3}, b\frac{a+b}{2}\right\}$
$m=1$	$\{e_1\}\{E, E\}$	M^1	$(2)(3)^1$
	$\{E\}\{e_1, e_1\}$	M^1	$(2)(3)^1$
	$\{e_1\}\{e_1, e_1\}$	M^1	$(2)(3)^1$
	$\{E\}\{E, e_1\}$	M^1	$(2)(4)^1$
	$\{e_1\}\{E, e_1\}$	M^1	$(2)(4)^1$
$m=2$	$\underline{\{e_1\}\{E, E\}}$		
	$\{e_1\}\{e_2, e_2\}$	M^2	$(2)(3)^2$
	$\{e_1e_2\}\{e_2, e_2\}$	M^2	$(2)(3)^2$
	$\{e_1\}\{E, e_2\}$	M^2	$(2)(4)^2$
	$\{e_1e_2\}\{E, e_2\}$	M^2	$(2)(4)^2$
	$\{e_1e_2\}\{E, E\}$	${}^2M$	
	$(2)(4)^1 \rightarrow 6M^2$		
$m=3$	$\underline{\{e_1\}\{e_2, e_2\}^+}$		
	$\{e_1\}\{e_2, e_2e_3\}$	M^3	
	$\{e_1e_3\}\{e_2, e_2e_3\}$	M^3	
	$(2)(4)^2 \rightarrow 4M^3$		

$N_1(Bm) = 5$

$N_2(Bm) = 2\underline{6} + 3\underline{4} = 24$
$N_3(Bm) = 18\underline{4} + 6\underline{2} = 84.$

For $m = 1$, the following groups correspond to the antisymmetric characteristics:

(1) $\left\{a, b, \frac{a+c}{2}\right\}(m_1)(Bm_1)$;

(2) $\left\{a, b, \left(\frac{a+c}{2}\right)_1\right\}(m)(B_{0,0,1}m)$;

(3) $\left\{a, b, \left(\frac{a+c}{2}\right)_1\right\}(m_1)(B_{0,0,1}m_1)$;

(4) $\left\{a, b_1, \frac{a+c}{2}\right\}(m)(B_{0,1,0}m)$;

(5) $\left\{a, b_1, \frac{a+c}{2}\right\}(m_1)(B_{0,1,0}m_1)$.

For $m = 2$, from the catalogue of antisymmetric characteristics, derived from the M^1-type (1) group with the $(2)(3)^1$ antisymmetric characteristic type, by

* The partial catalogue has been deposited with the British Library Document Supply Centre as Supplementary Publication No. SUP 43479 (111 pp.). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.

Table 2. *The numerical results for $N_m(G)$ and N_m numbers*

G	$N_1(G)$	$N_2(G)$	$N_3(G)$	$N_4(G)$	$N_5(G)$	$N_6(G)$
1*s*	1	1	1			
2*s*	2	4	8	15		
3*s*	5	28	168	840		
4*s*	4	15	42			
5*s*	5	34	266	1680		
6*s*	5	24	84			
7*s*	8	85	1148	17 220	208 320	
8*s*	9	84	756	5040		
9*s*	5	48	756	14 280	208 320	
11*s*	3	10	28			
12*s*	3	21	210	1680		
13*s*	11	186	3948	83 160	1 249 920	
14*s*	11	126	1344	10 080		
17*s*	9	108	1260	10 080		
18*s*	9	192	7028	316 260	13 749 120	419 973 120
19*s*	17	348	7812	166 320	2 499 840	
20*s*	7	58	504	3360		
21*s*	9	176	4424	104 160	1 666 560	
23*s*	3	6				
25*s*	7	42	168			
27*s*	2	3				
28*s*	8	75	714	5040		
37*s*	15	210	2520	20 160		
38*s*	1					
61*s*						
3*h*	7	54	420	2520		
5*h*	5	39	357	2520		
8*h*	3	9	21			
19*h*	4	19	98	420		
21*h*	15	306	7224	161 280	2 499 840	
1*a*	2	4	7			
8*a*	1	1				
21*a*	5	44	448	3360		
29*a*	3	14	56			

$N_1 = 1191$
$N_2 = 9511$
$N_3 = 109\,139$
$N_4 = 1\,640\,955$
$N_5 = 28\,331\,520$
$N_6 = 419\,973\,120$

the algorithm (Jablan, 1986*b*), the antisymmetric characteristics of the eight M^2-type groups [labelled (5)–(12)], derived from the (2), (3) M^1-type groups with the same $(2)(3)^1$ antisymmetric characteristic type, are obtained in the form of comparative catalogues of the M^2-type groups, which in international symbols are:

(1) $B_{0,0,10}m_1$ (5) $B_{0,0,11}m_{10}$ (1′) (9) $B_{0,0,11}m_1$ (1″)
(2) $B_{0,0,10}m_{11}$ (6) $B_{0,10,1}m$ (2′) (10) $B_{0,10,1}m_1$ (2″)
(3) $B_{0,10,0}m_1$ (7) $B_{0,10,1}m_{10}$ (3′) (11) $B_{0,10,1}m_{11}$ (3″)
(4) $B_{0,10,0}m_{11}$ (8) $B_{0,0,1}m_{10}$ (4′) (12) $B_{0,0,1}m_{11}$ (4″).

The remaining $12M^2$-type groups [labelled (13)–(24)] are obtained from the (4), (5) M^1-type groups with the $(2)(4)^1$ antisymmetric characteristic type, exclusively by respecting the criterion of independence of the antiidentities (Jablan, 1984, 1986*b*, Theorem 1*b*). All the cases of this type in the partial catalogue are marked by →.

(13) $B_{0,1,0}m_{10}$ (19) $B_{0,1,0}m_{11}$ (13′)
(14) $B_{0,11,10}m$ (20) $B_{0,11,10}m_1$ (14′)
(15) $B_{0,11,10}m_{10}$ (21) $B_{0,11,0}m_1$ (15′)
(16) $B_{0,11,0}m_{10}$ (22) $B_{0,11,10}m_{11}$ (16′)
(17) $B_{0,1,10}m$ (23) $B_{0,1,10}m_1$ (17′)
(18) $B_{0,1,10}m_{10}$ (24) $B_{0,1,10}m_{11}$ (18′).

By an analogous method, the complete catalogue of the M^3-type groups derived from the $Bm(6s)$ group which consists of $84M^3$-type groups can be obtained.

The numerical results for the $N_m(G)$ and N_m numbers are given in Table 2.

All the simple and multiple antisymmetric M^m-type space groups can be obtained from the partial catalogue according to the method and algorithm of partial cataloguing (Jablan, 1986*b*), or exclusively by using the criterion of independence of the $e_1, e_2, \ldots, e_m$ antiidentities (Jablan, 1984, 1986*b*, Theorem 1*b*) (cases marked by →).

By the application of the partial catalogue, which consists of 179 M^1-type groups, 638 M^2-type and 36 ${}^2M$-type groups, 726 M^3-type and 111 ${}^3M$-type groups, 403 M^4-type and 56 ${}^4M$-type groups, 110 M^5-type and 15 ${}^5M$-type groups and 16 M^6-type groups, the computation of $N_m(G)$ and N_m and the production of the complete catalogue, consisting of 450 065 436 groups (which exceeds the possibilities of computation) can be carried out.

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Electron-Microscope Imaging of Short-Range Order in Disordered Alloys

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Abstract

Dynamical diffraction effects of diffuse scattering due to point defects in crystals and the associated imaging are studied in relation to imaging of the short-range order in disordered alloys through theoretical formulations and computer image simulations. Simulations of gold crystals including one point defect without strain show that the image contrast is localized in atomic size and relatively insensitive to the defect depth when the total thickness is fixed. The contrast from a small number of defects in an atomic column can be simply related to the number of defects by a nonlinear expression. An approximate imaging theory for short-range order in disordered binary alloys is discussed. The present study shows that the important parameters for observations of defective crystals are total thickness and microscope defocus, but not the defect depth. With controlled values of these parameters and sample conditions, the images of disordered binary alloys can be interpreted semi-quantitatively.

1. Introduction

Electron-microscope observations of short-range order (SRO) in disordered alloys have been performed with diffuse scattering in the dark-field (Yamaguchi, Watanabe & Ogawa, 1961; Ruedl, Delavignette & Amelinckx, 1968; Chevalier & Stobbs, 1979) and bright-field (Dutkiewicz & Thomas, 1975) imaging modes. In the dark-field images, bright speckles about 2 nm in size were observed in various disordered alloys, which suggested small ordered domains. In the dark-field lattice images taken under conditions of diffuse scattering and fundamental reflections, the image features suggested localized ordered structures in a disordered matrix (Van Tendeloo, De Ridder & Amelinckx, 1978; Tanaka, Ohshima, Harada & Mihama, 1979). In the bright-field lattice images taken under conditions of direct wave and diffuse scattering, localized ordered regions were clearly observed with localized lattice-like fringes in disordered Cu–Pd (Tanaka & Ohshima, 1984; Van Tendeloo & Amelinckx, 1983) and Au–Mn alloys (Tanaka, Cowley & Ohshima, 1987).

The basis of image interpretation for disordered crystals in dark and bright fields is not sufficiently well established to allow analysis of the localized structures. The bright speckles in the dark-field images do not always correspond to single ordered domains (Cowley, 1973). The image features in the dark-field lattice images do not give correct information on atomic structures even for two-dimensional ordered alloys (Terasaki, Wood & Watanabe, 1984). In the axial bright-field images the overlap problem has to be clarified in the imaging theories and simulations.

In high-resolution electron microscopy of disordered or defective crystals, methods of intuitive image interpretation have to be sought, even though they are admitted to be just a first approximation. The standard method for the structure analysis, namely comparison between actual images and simulated ones, is not applicable to disordered crystals without starting models. A useful approximation for intuitive image interpretation is the weak-phase-object approximation, which is applicable to thin samples composed of light atoms. Other useful

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